Gap probabilities in piecewise thinned Airy and Bessel point processes

Christophe Charlier

KTH Royal Institute of Technology

cchar@kth.se

Based on several works with T. Claeys and A. Doeraene

Gap in the piecewise thinned Airy point process

The Airy point process is a determinantal point process on $\mathbb{R}$, arising near soft edges of certain large random matrices.

Let $m \in \mathbb{N}_{>0}$, $\vec{s} = (s_1, \dots, s_m) \in [0,1]^m$ and $\vec{x} = (x_1, \dots, x_m) \in \mathbb{R}^m$ be such that $-\infty < x_m < \dots < x_1 < x_0 = +\infty$.

For $j \in \{1, 2, \dots, m\}$, each particle on the interval (x_j, x_{j-1}) is removed with probability s_j.

We consider the probability to observe a gap on $(x_m, +\infty)$ in the thinned process. This probability can be written as a Fredholm determinant:

$$F(\vec{x};\vec{s}) = \det\left(1 - \chi_{(x_m,+\infty)} \sum_{j=1}^{m} (1 - s_j) K^{\mathrm{Ai}} \chi_{(x_j, x_{j-1})}\right).$$

Exact expression for $F(\vec{x};\vec{s})$ and a system of Painlevé II equations

If $m = 1$, Tracy and Widom ('94) have shown that $F(x;s)$ can be expressed in terms of a solution to a Painlevé II equation.

This result was generalised by Claeys-Doeraene ('18) for an arbitrary $m \geq 1$ as follows

$$F(\vec{x};\vec{s}) = \prod_{j=1}^{m} \exp\left(-\int_0^{+\infty} \xi q_j^2(\xi;\vec{x},\vec{s}) d\xi\right),$$

where $q_1, \dots, q_m$ satisfy a system of m coupled Painlevé II equations

$$q_j'' = (\xi + x_j) q_j + 2 q_j \sum_{\ell=1}^{m} q_\ell^2, \qquad j = 1, \dots, m,$$

$$q_j(\xi;\vec{x},\vec{s}) = \sqrt{s_{j+1} - s_j}\,\mathrm{Ai}(\xi + x_j)(1 + o(1)), \qquad \text{as } \xi \to +\infty,$$

where $s_{m+1} := 1$.